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Hybrid Reynolds-Averaged Navier–Stokes/Large Eddy Simulation Approach for Predicting Jet Noise

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Hybrid acoustic prediction methods have an important advantage over the current Reynolds-averaged Navier–Stokes based methods in that they only involve modeling of the relatively universal subscale motion and not the configuration-dependent larger-scale turbulence. Unfortunately, they are unable to account for the high-frequency sound generated by the turbulence in the initial mixing layers. This paper introduces an alternative approach that directly calculates the sound from a hybrid Reynolds-averaged Navier–Stokes/large eddy simulation flow model (which can resolve the steep gradients in the initial mixing layers near the nozzle lip) and adopts modeling techniques similar to those used in current Reynolds-averaged Navier–Stokes based noise prediction methods to determine the unknown sources in the equations for the remaining unresolved components of the sound field. The resulting prediction method would then be intermediate between the current noise prediction codes and previously proposed hybrid noise prediction methods.

Nomenclature

c	=	speed of sound
$\tilde{D}_0$	=	modified convective derivative
$D^{\mu}_{\nu j}$	=	spatial derivative operator
$\frac{D}{Dt}$	=	convective derivative
f	=	dummy variable
$g^{a}_{\nu\mu}$	=	adjoint vector Green's function
$g_{\nu\mu}$	=	vector Green's function
h	=	enthalpy
h_0	=	stagnation enthalpy
$K_{\mu\nu}$	=	component of linear Euler operator
$L_{\mu\nu}$	=	linear Euler operator
p	=	pressure
$\hat{p}$	=	component of pressure determined by base flow
R	=	ideal gas constant
$R_{\sigma i \mu j}$	=	residual stress tensor
s_{μ}	=	residual source vector
T	=	absolute temperature, large time interval
t	=	time
u_i	=	generalized residual velocity variable
V	=	integration over all space
$\boldsymbol{v}$	=	velocity vector
v_i	=	component of $\boldsymbol{v}$, $i = 1, 2, 3$
$\boldsymbol{x}$	=	position vector (usually associated with the observation point in the Green's function formulas)
x_i	=	Cartesian component of $\boldsymbol{x}$, $i = 1, 2, 3$
$\boldsymbol{y}$	=	position vector of source
y_i	=	Cartesian component of $\boldsymbol{y}$, $i = 1, 2, 3$
$\Gamma_{j\sigma l\mu}$	=	correlation of instantaneous propagator
γ	=	specific heat ratio
$\gamma_{j\mu}$	=	instantaneous propagator
$\tilde{\gamma}_{\nu j\mu}$	=	instantaneous propagator
$\delta_{\mu\nu}$	=	five-dimensional Kronecker delta
$\delta(\cdot)$	=	Dirac delta function
$\boldsymbol{\eta}$	=	separation vector in residual stress correlations
θ_{ij}	=	base flow stress tensor
∂_{ν}	=	generalized spatial derivative
Π''	=	far-field pressure autocovariance
π	=	generalized residual pressure variable
ρ	=	density
$\tilde{\sigma}_{\mu j}$	=	base flow stress tensor
$\sigma'_{\mu j}$	=	generalized Reynolds stress tensor
τ	=	source point time variable
$\hat{\cdot}$	=	base flow quantity, filtered quantity
$\tilde{\cdot}$	=	base flow quantity, Favre-filtered quantity
$\bar{\cdot}$	=	time average
$\langle\cdot\rangle$	=	arbitrary spatial filter

Superscripts

$'$	=	residual quantity, dummy integration variable
$''$	=	component of residual motion driven by residual stress, dummy integration variable, and fluctuating quantity

I. Introduction

CURRENT noise prediction methods are usually based on acoustic analogy approaches [1–3] that (as a minimum) require modeling the entire unsteady flow that actually produces the sound and are therefore unlikely to be universal beyond the data bases for which they have been calibrated. This difficulty would not occur if the sound field were determined from a direct Navier–Stokes (DNS) solution [4] or even an ordinary large eddy simulation (LES). Unfortunately, the computer resources needed to resolve all the relevant length scales are enormous, often well beyond those available on present-day machines. A reasonable compromise would be to use LES simulations with very broad filter widths. This approach was adopted by Bastin, Lafon and Candel [5], Bodony and Lele [6–8], and others. They found that simulations of this type can produce reasonable results at low frequencies, but usually at the expense of the high-frequency component of the spectrum, which they tend to significantly underpredict. This probably occurs because these methods do not account for the sound generated by the unresolved turbulence scales, which can correspond to the entire unsteady flow in the thin shear layers near the nozzle lip [9,10] as well as to the subfilter-scale motion further downstream [8,10]. The present paper is an attempt to resolve these difficulties by directly calculating the low-frequency sound from a hybrid RANS/LES computation (that can resolve the thin shear layers near the nozzle lip) and determining the remaining small-scale, or high-frequency,

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sound by modeling the source terms in an appropriate set of linear equations.

It is expected that the hybrid RANS/LES computations will be based on the usual conservation law type equations with stress and heat flux models that vary continuously from RANS type models near the nozzle lip to full LES-type models further downstream [11]. But, unlike the related detached eddy type approaches, the appropriate model choice will most likely be determined by resolution requirements and not by the distance from a wall. The equations for the remaining higher-frequency sound are obtained by dividing each of the flow variables in the Navier–Stokes equations into a *base flow* component that satisfies universal set of hyperbolic conservation laws (the RANS/LES equations in the present approach) which include, among other things, the Navier–Stokes equations themselves, the Reynolds-averaged Navier–Stokes equations as well as the Favre-filtered [12] Navier–Stokes equations with arbitrary closure models for the subfilter-scale stresses (i.e., the equations that are actually used in most LES calculations) and a *residual* component that satisfies the Navier–Stokes equations with the base flow equations subtracted out. By introducing new (in general, nonlinear) dependent variables [13,14] the latter equations (which are at this point exact) can be rewritten in the form of the linearized Euler equations with sources that are formally the same as those that would be produced by external stress and heat flux perturbations. The corresponding source strengths, which depend on the nonlinear residual Reynolds stresses, can, in principle, be modeled, and the resulting linear equations can be solved by using a Green's function approach. The result can then be used to obtain an expression for the far-field pressure autocovariance in terms of the residual-scale turbulent motion. The latter would only depend on the correlation (i.e., the lower-order statistics) of this motion if the base flow were taken to be a steady RANS solution. But the result for any time-dependent base flow, such as an LES solution to the filtered Navier–Stokes (FNS) equations, appears to depend on the detailed *instantaneous* residual-scale stresses, which are very difficult to model.

Current RANS-based noise prediction codes, such as the JeNo [15] code, determine the unknown source strengths from experimental data. This cannot be done when the instantaneous source strengths have to be modeled, because, as a practical matter, experimentalists are limited to documenting the reproducible (i.e., nonrandom) characteristics of these sources, such as their correlations or other lower-order statistics. It is therefore important to derive a formula for the residual sound field that depends only on nonrandom quantities the can be measured with currently available experimental techniques. The present paper is an attempt to obtain such an equation by exploiting the statistical independence (i.e., the decoupling) of any base flow solution (such as a hybrid RANS/LES simulation) from the detailed subscale fluctuations that would occur in an actual experiment. The only coupling that can occur in these simulations is through the base flow stresses and heat flux vector, which are usually modeled in terms of the remaining base flow variables and their derivatives in hybrid RANS/LES computations. The base flow solutions are therefore calculated from a closed set of equations that involve only base flow variables. The chaotic subscale fluctuations in any realization of the flow (i.e., in an actual experiment) are therefore unlikely to be correlated with the fluctuations in these solutions.

The present paper assumes that the model for the residual (i.e., unresolved) stresses (say, $\sigma'_{\mu j}$) is constructed from appropriately reduced experimental data (which can, in principle, be obtained from particle imaging velocimetry measurements). Although the actual (i. e., the experimental) resolved and residual motions are certainly correlated, the experimentally based model for $\sigma'_{\mu j}$ should reflect the fact that the experimental residual-scale motions are expected to be statistically independent of the fluctuations in the base flow (i.e., the hybrid RANS/LES) solution and should therefore be uncorrelated with the fluctuations in that solution.

As already indicated, the present paper introduces a hybrid approach in which the base flow goes from a slightly unsteady RANS type (URANS) solution near the nozzle lip (where the initial shear layers are too thin to be resolved by the necessarily coarse LES computation [9,10]) to a (fairly large filter width) LES solution further downstream. We adopt the so-called universal modeling approach [11], which assumes that the unknown stresses and heat flux in the base flow conservation laws are determined by a hybrid model that goes from an ensemble average (i.e., RANS or URANS) type model near the nozzle lip to a pure spatial LES-type model further downstream. Maintaining conservation law form seems to be very desirable at the large grid sizes being envisioned in the present approach [16]. Fortunately, there is no need to sacrifice this property by requiring that the base flow satisfy the (space-time) filtered Navier–Stokes equations with inhomogeneous filtering as was done, for example, by Germano [17], because the combined result (i.e., the base flow plus residual equation solution) is, in principle, *exact*. But the subscale stress correlations in the residual pressure equation (that go continuously from almost the entire fluctuating stress near the nozzle lip to the instantaneous subfilter-scale stresses further downstream) ultimately have to be modeled.

At high Reynolds numbers, the small-scale motion is expected to be statistically independent of large energy-bearing scales in any realization of the flow. This does not, however, imply that all of the resolved motion will be statistically independent of the subscale motion, because the near cutoff scales are expected to be highly correlated. It is only the hybrid RANS/LES simulation of the resolved scales that is expected to be uncorrelated with the actual (i. e., the experimentally measured) subscale tensor $\sigma'_{\mu j}$. But the propagation factor in the Green's function solution is likely to be dominated by the large energy-bearing scales, and Kolmogorov's [18] hypothesis (which forms the bases of many of the current subscale turbulence models [11]) indicates that the small-scale motion should be statistically independent of these scales. It is, therefore, likely that the present decorrelation assumption will be satisfied, at least on a global basis, in the downstream region of the actual flow (where the implied filter width eventually becomes small relative to the transverse length scale).

Because the covariance of any steady quantity with any other quantity is identically zero, the statistical independence assumption should also be approximately satisfied in the upstream region, where the base flow satisfies the RANS (or URANS) equations and is nearly steady. All of the unsteadiness in this nearly parabolic region of the flow must force its way upstream from the downstream LES region. This statistical independence assumption may not, however, be satisfied in the intermediate blending region. But to the extent that the propagation factor is nonlocal and the blending region is sufficiently small, the present decorrelation assumption should be reasonably well satisfied in any actual realization of the flow.

A major attraction of the current hybrid approach is that the subscale stresses that have to be modeled in the downstream region should be much more universal than the large-scale stresses that have to be modeled with the usual RANS-based methods. Unfortunately, the present approach still requires modeling nearly all of the unsteady flow in the initial mixing layers. But it is now believed that, whereas the sound produced by the larger-scale motion at the end of the potential core is relatively coherent, the relatively high-frequency sound produced by the small-scale mixing layer motions (as well as by the smaller-scale motion beyond the end of the potential core) tends to be much more random [19]. The latter should therefore be less sensitive to variations in retarded time than the former, which implies that it should also be less sensitive to the details of the source structure and, consequently, that relatively universal source models can be constructed. The lower-frequency sound seems to be reasonably well predicted by existing LES codes.

The present paper provides a rational framework for combining an LES technique that directly calculates the latter with a RANS-based methodology that models the former. But, unlike existing RANS-based approaches, the present formalism explicitly accounts for the scattering of the small-scale sound by the large-scale motion: an effect that was emphasized by Crighton [2]. To the extent that this phenomenon is nonlocal, the underlying statistical independence assumption should be appropriate for its computation.

As noted previously, we envision using conventional LES-type techniques to directly calculate the sound from more or less standard hybrid RANS/LES equations (see Chapter 11 of Sagaut [11]). The focus of this paper is, therefore, on determining the remaining higher-frequency (or residual) component of the sound field. The appropriate equations are set out in Sec. II. Because these equations turn out to be formally linear (with the true nonlinearity hidden in the nonlinear form of the dependent variables and the unknown source terms), the powerful techniques of linear analysis can be used to construct their solution. This property is first used to decompose the solution into the sum of two independent components that are determined by separate (i.e., uncoupled) sets of linear equations. The first component, which describes the larger of the unresolved (i.e., residual) scales of motion, is determined by equations with base-flow-dependent coefficients and source terms and can, therefore, be computed right along with the base flow solution with very little additional effort. The second component is determined by an equation set whose coefficients depend on the base flow solution but contain source terms that depend on the unknown residual stresses (that ultimately have to be modeled). The remainder of the paper is, therefore, devoted to calculating this component.

The linearity of its governing equations is again exploited (in Sec. III) to construct a formal vector Green's function solution that separates the base flow and residual components of the motion. The result is then used in Sec. IV to obtain a formula relating the far-field pressure autocovariance to the (unknown) time-dependent residual stresses. The determination of the sound field is, therefore, reduced to the determination of these unknown stresses. But, as already indicated, predicting the instantaneous values of this random stress field is an enormously difficult task. This sets the stage for exploiting the statistical independence arguments to obtain a result that only depends on the correlation of these stresses and not on their instantaneous values, which, as noted previously, greatly simplifies the requisite modeling. The detailed steps are described in Sec. IV.

II. Subfilter-Scale Equations in Vector Form

We decompose the pressure p, density ρ, and velocity v_i, $i = 1, 2, 3$ into base flow components $\hat{\rho}$, $\hat{p}$, and $\tilde{v}_i$ and residual components defined by

$$\rho' = \rho - \hat{\rho}, \qquad v_i' = v_i - \tilde{v}_i, \qquad p' = p - \hat{p} \tag{1}$$

where the base flow components satisfy a universal set of hyperbolic conservation laws, which include, among other things, the Navier–Stokes equations themselves, the RANS equations as well as the Favre-filtered [12] Navier–Stokes equations (in which case $\hat{\rho}$, $\hat{p}$, and

$$\tilde{v}_i \equiv \widehat{\rho v_i}/\hat{\rho}$$

would correspond to filtered and Favre-filtered quantities, respectively). The residual components can be shown to satisfy the five formally linear equations [13,14]

$$L_{\mu\nu} u_\nu = s_\mu \quad \text{for } \mu, \nu = 1, 2, 3, 4, 5 \tag{2}$$

where

$$\{u_\nu\} \equiv \{\rho v_i', \pi, \rho'\} \equiv \{\rho v_1', \rho v_2', \rho v_3', \pi, \rho'\}, \qquad \pi \equiv p' - \tfrac{(\gamma-1)}{2}\sigma_{jj}' \tag{3}$$

denotes the solution vector and $L_{\mu\nu}$ is defined by

$$L_{\mu\nu} \equiv \delta_{\mu\nu}\tilde{D}_0 + \delta_{\nu 4}\partial_\mu + \partial_\nu(\widetilde{c^2}\delta_{\mu 4} + \delta_{\mu 5}) + K_{\mu\nu} \tag{4}$$

with

$$K_{\mu\nu} \equiv \partial_\nu \tilde{v}_\mu - \frac{1}{\bar{\rho}}\frac{\partial\tilde{\theta}_{\mu j}}{\partial x_j}\delta_{\nu 5} + (\gamma - 1)\left(\frac{\partial \tilde{v}_j}{\partial x_j}\delta_{\nu 4} - \frac{1}{\bar{\rho}}\frac{\partial\tilde{\theta}_{\nu j}}{\partial x_j}\right)\delta_{\mu 4} \tag{5}$$

$i, j = 1, 2, 3$

and

$$\tilde{D}_0 \equiv \frac{\partial}{\partial t} + \frac{\partial}{\partial x_j}\tilde{v}_j; \qquad \tilde{\theta}_{ij} \equiv \delta_{ij}\bar{p} - \tilde{\sigma}_{ij}; \qquad \partial_\mu \equiv \frac{\partial}{\partial x_j}$$
$$\mu = j = 1, 2, 3; \qquad \tilde{v}_\mu, \tilde{\theta}_{\mu j}, \partial_\mu = 0, \qquad \mu = 4, 5 \tag{6}$$

The source function s_μ is defined by

$$s_\mu \equiv D^\mu_{\nu j}(\sigma_{\nu j}' - \tilde{\sigma}_{\nu j}) \tag{7}$$

with

$$D^\mu_{\nu j} \equiv \delta_{\nu\mu}\frac{\partial}{\partial x_j} + \delta_{4\mu}(\gamma - 1)\frac{\partial \tilde{v}_\nu}{\partial x_j} \tag{8}$$

and

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \tilde{v}_j\frac{\partial}{\partial x_j}$$

where

$$\sigma_{\mu j}' \equiv -\rho v_\mu' v_j' + \tfrac{\gamma-1}{2}\delta_{\mu j}\rho v'^2, \qquad v_4' \equiv (\gamma - 1)h_0', \qquad v_5' = 0 \tag{9}$$

The fifth component $\tilde{\sigma}_{5j}$ of $\tilde{\sigma}_{\nu j}$ is zero. The remaining components $\tilde{\sigma}_{ij}$ and $\tilde{\sigma}_{4j}$ are the base flow stresses and heat flux vector, which would be given by

$$\tilde{\sigma}_{ij} \equiv -\hat{\rho}(\widetilde{v_i v_j} - \tilde{v}_i\tilde{v}_j) + \tfrac{\gamma-1}{2}\delta_{ij}\hat{\rho}(\widetilde{v^2} - \tilde{v}^2)$$

$$\tilde{\sigma}_{4j} \equiv -(\gamma - 1)[\hat{\rho}(\widetilde{h_0 v_j} - \tilde{h}_0\tilde{v}_j) + \tfrac{1}{2}\tilde{\sigma}_{ii}\tilde{v}_j + \tilde{\sigma}_{ij}\tilde{v}_i]$$

if the base flow were chosen to be the FNS equations. Finally,

$$h_0 \equiv h + \tfrac{1}{2}v^2$$

and

$$h_0' \equiv h' + \tfrac{1}{2}v'^2$$

where

$$\widetilde{c^2} = \gamma R\tilde{T}$$

is the squared base flow sound speed and, as usual, the viscous terms, which are believed to play an insignificant role in the sound generation process, have been neglected. The Latin indices go from one to three, whereas the Greek indices go from one to five.

Equations (7–9) show that the source term in Eq. (2) consists of two parts: a part σ_{ij}' associated with the residual (or subscale) motion and a base flow component $\tilde{\sigma}_{ij}$ that is the same as the source that appears in the hybrid RANS/LES equations. The subscale motion generated by this latter source component is completely determined by the base flow solution, because it is determined by Eq. (2), whose coefficients depend on that solution, whereas the subscale motion generated by σ_{ij}' is determined by the base flow solution as well as by the experimentally measured residual motion and is, therefore, correlated with both these entities.

Then, because Eq. (2) is linear, it makes sense to divide its solution vector into the two components

$$u_\nu = u_\nu' + \tilde{u}_\nu \tag{10}$$

where

$$L_{\mu\nu}\tilde{u}_\nu = -D^\mu_{\lambda j}\tilde{\sigma}_{\lambda j} \tag{11}$$

and

$$L_{\mu\nu}u_\nu' = D^\mu_{\lambda j}\sigma_{\lambda j}' \tag{12}$$

It is clear that these results will remain unchanged if σ_{ij}' and $\tilde{\sigma}_{ij}$ are redefined by

$$\sigma'_{\mu j} \to -\rho v'_\mu v'_j + \frac{\gamma - 1}{2}\delta_{ij}\rho v'^2 - \bar{\sigma}_{\mu j}(\boldsymbol{x}) \tag{13}$$

$$\tilde{\sigma}_{\mu j} \to \tilde{\sigma}_{\mu j} - \bar{\sigma}_{\mu j}(\boldsymbol{x}) \tag{14}$$

where $\bar{\sigma}_{\mu j}(\boldsymbol{x})$ is an arbitrary time-independent quantity. The overbar will now be used more specifically to denote time (or better ensemble) averages. Then the time average of the unmodified $\sigma'_{\mu j}$ [defined be Eq. (9)] would nearly equal the time average of the unmodified $\tilde{\sigma}_{\mu j}$ in the upstream region, where the base flow is almost steady. (They would be exactly equal if the flow were completely steady.) Appendix B shows that this will also be approximately true in the downstream region, where the original $\tilde{\sigma}_{\mu j}$ corresponds to the subgrid stresses and heat flux in the FNS equations and are given in the paragraph following Eq. (9). We, therefore, take $\bar{\sigma}_{\mu j}(\boldsymbol{x})$ to be the (approximate) common value of these two quantities and thereby insure that the derived stresses [Eqs. (13) and (14)] have nearly zero time averages and, more important, that $\tilde{\sigma}_{\mu j}$ is almost equal to zero in the upstream region, where the base flow is nearly steady. The solution to Eq. (10) is, therefore, almost entirely driven by the initial conditions in this region. Notice that steady sources cannot generate any sound at subsonic Mach numbers.

Because the coefficients and sources in Eq. (11) are all resolved quantities (i.e., they involve only variables that appear in the base flow equations), it can be solved right along with these equations with no additional modeling and little additional effort. These two sets of equations [i.e., Eq. (11) and the base flow equations] form a closed system once the base flow stress model has been selected. The pressure components of their solution vectors (say, $\hat{p}'$ and $\hat{p}$) can then be added together to obtain the component of the pressure

$$\hat{p}'' \equiv \hat{p}' + \hat{p} \tag{15}$$

that is completely determined by the base flow (i.e., the hybrid RANS/LES) solution. It may even be possible to calculate these solutions on the same mesh, because $\tilde{u}_v$ is expected to be composed of the larger subgrid scales [6]. But even if this is not the case, the solution to the linear system [Eq. (11)] should still be relatively inexpensive compared with the base flow solution.

The solution to Eq. (11) can also be used to iteratively calculate at least a portion of the modeled stresses in the hybrid RANS/LES equations in order to gradually introduce the required unsteadiness into the downstream LES solution. This type of calculation, which is sometimes known as *mean field theory*, should only be required in the blending region between the pure RANS and LES regions. It is expected to provide a better representation of the actual physics than current techniques for inputting unsteadiness into LES computations.

III. Vector Green's Function Solution for π'

Solving Eq. (12) is potentially much more difficult than solving Eq. (11), because of the additional source modeling requirement and because its solution can depend upon both the base flow and residual motions. To separate out these effects, we introduce the vector Green's function [20]

$$g_{\nu\sigma}(\boldsymbol{x}, t|\boldsymbol{y}, \tau)$$

which satisfies the inhomogeneous linear equations

$$(L_{\mu\nu})_{x,t}\, g_{\nu\sigma}(\boldsymbol{x}, t|\boldsymbol{y}, \tau) = \delta_{\mu\sigma}\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - \tau) \tag{16}$$

with delta-function type source term together with the causality condition

$$g_{\nu\mu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau) = 0 \quad \text{for } t < \tau \tag{17}$$

to obtain

$$\begin{aligned} u'_\nu(\boldsymbol{x}, t) &= \int_V \int_{-\infty}^{\infty} g_{\nu\mu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau) D^\mu_{\lambda j}\sigma'_{\lambda j}\, \mathrm{d}\boldsymbol{y}\, \mathrm{d}\tau \\ &= -\int_V \int_{-\infty}^{\infty} \tilde{\gamma}_{\nu j\mu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau)\sigma'_{\mu j}(\boldsymbol{y}, \tau)\, \mathrm{d}\boldsymbol{y}\, \mathrm{d}\tau \end{aligned} \tag{18}$$

where

$$\tilde{\gamma}_{\nu j\mu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau) \equiv \frac{\partial g_{\nu\mu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau)}{\partial y_j} - (\gamma - 1)\frac{\partial \tilde{v}_\mu}{\partial y_j} g_{\nu 4}(\boldsymbol{x}, t|\boldsymbol{y}, \tau) \tag{19}$$

From the acoustics perspective, the primary interest is in the fourth pressurelike component of Eq. (18), which only involves the fourth component vector Green's function:

$$g_{4\nu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau)$$

The latter quantity can, in principle, be found by solving the system [Eq. (16)]. But because this consists of 25 first-order equations, it turns out to be simpler to compute the adjoint Green's function:

$$g^a_{\nu 4}(\boldsymbol{y}, \tau|\boldsymbol{x}, t)$$

Equations (A1–A4) show that it is related to

$$g_{4\nu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau)$$

by the reciprocity relation

$$g^a_{\nu 4}(\boldsymbol{y}, \tau|\boldsymbol{x}, t) = g_{4\nu}(\boldsymbol{x}, t|\boldsymbol{y}, \tau) \tag{20}$$

and satisfies the fifth-order system

$$\left(L^a_{\mu\nu}\right)_{\boldsymbol{y},\tau} g^a_{\nu 4}(\boldsymbol{y}, \tau|\boldsymbol{x}, t) = \delta_{\mu 4}\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - \tau) \tag{21}$$

which is given in component form by Eqs. (A5a–A5c). These equations suggest that

$$\breve{g}_{\nu 4}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t - \tau) \equiv g^a_{\nu 4}(\boldsymbol{y}, \tau|\boldsymbol{x}, t)$$

should be a stationary random function of its third argument τ, which results from the time dependence of the coefficients in

$$(L^a_{\mu\nu})_{y,\tau}$$

that are determined from the base flow (i.e., the hybrid RANS/LES) solution and are, therefore, themselves random functions of time [21]. (Notice that this argument would not appear if the base flow were steady.)

It therefore follows from Eqs. (3), (19), and (20) that the fourth component of Eq. (18) can be written as

$$\pi'(\boldsymbol{x}, t) \equiv u'_4 = -\int_V \int_{-\infty}^{\infty} \gamma_{j\mu}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t - \tau)\sigma'_{\mu j}(\boldsymbol{y}, \tau)\, \mathrm{d}\boldsymbol{y}\, \mathrm{d}\tau \tag{22}$$

where, like

$$\breve{g}_{\nu 4}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t - \tau) \equiv g^a_{\nu 4}(\boldsymbol{y}, \tau|\boldsymbol{x}, t)$$

the propagator

$$\gamma_{j\mu}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t - \tau) \equiv \begin{cases} \lambda_{jk}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t - \tau) & \mu = k = 1, 2, 3 \\ \frac{\partial g^a_{44}(\boldsymbol{y}, \tau|\boldsymbol{x}, t)}{\partial y_j} & \mu = 4 \end{cases} \tag{23}$$

should be random functions of its third argument τ, because the third arguments of the six independent components

$$\begin{aligned} \lambda_{jk}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t - \tau) \equiv{}& \frac{1}{2}\left[\frac{\partial g^a_{k4}(\boldsymbol{y}, \tau|\boldsymbol{x}, t)}{\partial y_j} + \frac{\partial g^a_{j4}(\boldsymbol{y}, \tau|\boldsymbol{x}, t)}{\partial y_k}\right] \\ & - \frac{(\gamma - 1)}{2}\left(\frac{\partial \tilde{v}_k}{\partial y_j} + \frac{\partial \tilde{v}_j}{\partial y_k}\right) g^a_{44}(\boldsymbol{y}, \tau|\boldsymbol{x}, t) \end{aligned} \tag{24}$$

of

$$\lambda_{jk}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t-\tau)$$

(which arise from the third argument of

$$\breve{g}_{\nu 4}(\boldsymbol{y}|\boldsymbol{x}{:}\tau, t-\tau)$$

as well as the τ dependence of $\partial\tilde{v}_k/\partial y_j$) also have this property.

IV. Subscale Pressure Autocovariance

Because $\pi(\boldsymbol{x}, t) \to p'(\boldsymbol{x}, t)$ as $x \to \infty$, the component π' of π driven by the residual stresses goes to the component p'' of the far-field pressure driven by those stresses and the autocovariance

$$\overline{\Pi''(\boldsymbol{x},\tau)} \equiv \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} p''(\boldsymbol{x},t)p''(\boldsymbol{x},t+\tau)\,\mathrm{d}t \quad \text{for large } |\boldsymbol{x}| \tag{25}$$

of that quantity can, therefore, be written as

$$\bar{\Pi}'' = \frac{1}{2T}\int_{-T}^{T}\int_{-\infty}^{\infty}\iint_V \Gamma_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t'', \tau+t'')\sigma'_{\sigma i}(\boldsymbol{y},t')\sigma'_{\mu j}(\boldsymbol{y}+\boldsymbol{\eta}, t'')\,\mathrm{d}\boldsymbol{y}\,\mathrm{d}\boldsymbol{\eta}\,\mathrm{d}t'\,\mathrm{d}t'' = \frac{1}{2T}\int_{-T}^{T}\int_{-\infty}^{\infty}\iint_V \Gamma_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t''+t', \tau+t''-t')\sigma'_{\sigma i}(\boldsymbol{y},t')\sigma'_{\mu j}(\boldsymbol{y}+\boldsymbol{\eta}, t''+t')\,\mathrm{d}\boldsymbol{y}\,\mathrm{d}\boldsymbol{\eta}\,\mathrm{d}t'\,\mathrm{d}t'' \tag{26}$$

where the new propagator

$$\Gamma_{j\sigma l\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t'', \tau+t''-t') \equiv \int_{-\infty}^{\infty}\gamma_{j\sigma}(\boldsymbol{y}|\boldsymbol{x}{:}t', t+\tau-t')\gamma_{l\mu}(\boldsymbol{y}+\boldsymbol{\eta}|\boldsymbol{x}{:}t'', t-t'')\,\mathrm{d}t = \int_{-\infty}^{\infty}\gamma_{j\sigma}(\boldsymbol{y}|\boldsymbol{x}{:}t', t+\tau+t''-t')\gamma_{l\mu}(\boldsymbol{y}+\boldsymbol{\eta}|\boldsymbol{x}{:}t'', t)\,\mathrm{d}t \tag{27}$$

is expected to be a stationary random function of its fourth and fifth arguments, because the original propagators

$$\gamma_{j\sigma}(\boldsymbol{y}|\boldsymbol{x}{:}t', t+\tau+t''-t')$$

and

$$\gamma_{l\mu}(\boldsymbol{y}+\boldsymbol{\eta}|\boldsymbol{x}{:}t'', t)$$

have that property relative to their corresponding arguments. It follows that

$$\Gamma_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t''+t', \tau+t'')$$

and the source function

$$R_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}, t'', t') \equiv \sigma'_{\sigma i}(\boldsymbol{y}, t')\sigma'_{\mu j}(\boldsymbol{y}+\boldsymbol{\eta}, t''+t') \tag{28}$$

are both stationary random functions of t'.

It seems reasonable to suppose that these two quantities are uncorrelated (relative to this variable) in the present approach, because the base flow solution, which is determined from a closed set of equations that only involve the base flow variables, is expected to be statistically independent of (i.e., decoupled from) the chaotic subscale motion in the actual experiment used to model these quantities. (In fact, the entire chaotic component of the experimental flow should tend to become decorrelated from the hybrid LES/RANS simulation, in which the randomness comes from the numerical round-off errors and/or the random initial conditions, which are themselves uncorrelated with the initial conditions in the experiment [22,23]). It is therefore reasonable to require that any experimentally based model used to represent $\sigma'_{\mu j}$ be statistically independent of the base flow motion (and, consequently, with the coefficients of $L^a_{\mu\nu}$).

This means (by definition) that the covariance

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\Gamma''_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t''+t', \tau+t'')R''_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}, t'', t')\,\mathrm{d}t' \tag{29}$$

where

$$\Gamma''_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t''+t', \tau+t'') \equiv \Gamma_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t''+t', \tau+t'') - \bar{\Gamma}_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t'', \tau+t'') \tag{30}$$

$$R''_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}, t'', t') \equiv R_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}, t'', t') - \bar{R}_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}, t'') \tag{31}$$

$$\bar{\Gamma}_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t'', \tau+t'') \equiv \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\Gamma_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t', t''+t', \tau+t'')\,\mathrm{d}t' \tag{32}$$

and

$$\bar{R}_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}, t'') \equiv \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}R_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}, t'', t')\,\mathrm{d}t' \tag{33}$$

must be identically zero. Using this result in Eq. (26) shows that

$$\overline{\Pi''(\boldsymbol{x},\tau)}$$

can be written as an integral

$$\overline{\Pi''(\boldsymbol{x},\tau)} = \int_{-\infty}^{\infty}\iint_V \bar{\Gamma}_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t'', \tau+t'')\bar{R}_{\sigma i\mu j}(\boldsymbol{y};\boldsymbol{\eta}t'')\,\mathrm{d}\boldsymbol{y}\,\mathrm{d}\boldsymbol{\eta}\,\mathrm{d}t'' \tag{34}$$

of the product of two nonrandom functions.

The first of these functions, which can be expressed as the integral

$$\bar{\Gamma}_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t'', \tau+t'') = \int_{-\infty}^{\infty}\bar{\gamma}_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t'', t+\tau+t'', t)\,\mathrm{d}t \tag{35}$$

over time of the correlation

$$\bar{\gamma}_{i\sigma j\mu}(\boldsymbol{y},\boldsymbol{\eta}|\boldsymbol{x}{:}t'', t+\tau+t'', t) \equiv \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\gamma_{i\sigma}(\boldsymbol{y}|\boldsymbol{x}{:}t', t+\tau+t'')\gamma_{l\mu}(\boldsymbol{y}+\boldsymbol{\eta}|\boldsymbol{x}{:}t'+t'', t)\,\mathrm{d}t' \tag{36}$$

of the two random propagators

$$\gamma_{i\sigma}(\boldsymbol{y}|\boldsymbol{x}{:}t', t+\tau+t''-t')$$

and

$$\gamma_{l\mu}(\boldsymbol{y}+|\boldsymbol{x}{:}t'', t)$$

can be thought of as an expected, or mean, propagator. It can be determined as part of the hybrid LES/RANS computation. The second function, which can be thought of as the source function, describes the low-order statistics of the unknown component of the residual fluctuations. It has to be modeled, but it should be much easier to do this than to model the instantaneous values of these fluctuations (which would have to be done if the source and propagation fluctuations were not decorrelated). Notice that this decorrelation only implies that the fluctuation $\Gamma''_{i\sigma j\mu}$ (and not $\bar{\Gamma}_{i\sigma j\mu}$ itself) is independent of the unresolved scales. It may, therefore, be appropriate to parameterize the model for these scales (i.e., the functional form inferred from the experimental data) and determine the parameters from the hybrid RANS/LES solution.

We can also think of $\hat{p}$, $\tilde{v}_i$, and $\hat{\rho}$ and p', v'_i, and ρ' as the filtered and unfiltered components of the pressure, velocity, and density in the downstream region of an actual flow. Then the adjoint Green's function, and therefore the propagation factor $\Gamma_{i\sigma j\mu}$, should be dominated by the large energy-bearing scales of that flow, whereas Kolmogorov's [18] hypothesis (which forms the bases of many of the current subscale turbulence models [11]) indicates that the small-scale motion should be statistically independent of these scales. It is, therefore, likely that the present decorrelation assumption will be satisfied, at least on a global basis, in the downstream region of that flow (where the implied filter width eventually becomes small relative to the transverse length scale).

The statistical independence assumption should also be reasonably well satisfied in the upstream region, where the base flow satisfies the RANS equations and is nearly steady, because the fluctuating component of that flow and, therefore, the covariance (29) is almost identically zero. It may not, however, be satisfied in the intermediate blending region. But to the extent that the propagation factor is nonlocal and the blending region is sufficiently small, this should have relatively little effect on the predicted sound field.

V. Conclusions

An exact equation for the sound generation by the residual turbulence scales in a hybrid RANS/LES-based simulation [13,14] is used to obtain a formula for the residual-scale acoustic pressure autocovariance in terms of the residual-scale turbulence correlation tensor by exploiting the statistical independence of the hybrid RANS/LES solution and the experimentally determined residual motion. The residual acoustic radiation can therefore be calculated by introducing appropriate models for this relatively universal correlation while determining the configuration-dependent large-scale sound directly from the hybrid RANS/LES solution.

The approach also accounts for the scattering of the small-scale sound by the large-scale motion, an effect that was emphasized by Crighton [2]. To the extent that this phenomenon is nonlocal, the present statistical independence assumption should be very appropriate to its computation.

At high Reynolds numbers, the small-scale motion should be statistically independent of the large energy-bearing scales in any realization of the flow. But this does not imply that the resolved scales in the downstream LES region (i.e., the scales larger than the filter width) are all statistically independent of the subscale motion, especially near the filter cutoff. It is only the hybrid RANS/LES simulation of the filtered scales that is expected to be uncorrelated with the actual, that is, the experimentally measured, residual-scale tensor $\sigma'_{\mu j}$. But to the extent that the propagation factor $\Gamma_{i\sigma j\mu}$ is nonlocal and dominated by the large energy-bearing scales, Kolmogorov's [18] hypothesis that the small-scale motion is statistically independent of these scales suggests that the present decorrelation assumption will be satisfied in any actual realization of the flow. The hope is that the resulting Eq. (34) will accurately predict the subscale pressure autocovariance in that flow to the same extent as the hybrid RANS/LES solution predicts its large-scale statistics.

A major attraction of the current hybrid approach is that the subscale stresses that have to be modeled in the downstream region should be much more universal than the large-scale Reynolds stresses that have to be modeled with the usual RANS-based methods. Unfortunately, the present approach still requires modeling nearly the entire unsteady flow in the initial mixing layers. The hope is that it will be possible to obtain analytically based models for this region that exhibit a fair degree of universality.

Various extensions and modifications of this approach are, of course, possible. It can, for example, be generalized to include upstream nozzle effects by replacing the free space Green's function with one that satisfies appropriate no-slip boundary conditions on the nozzle walls. A major attraction of this idea is that it accommodates the imposition of more realistic internal boundary conditions, which may be a necessary step in achieving true noise suppression predictive capability [9].

Appendix A: Adjoint Vector Green's Function

The adjoint Green's function

$$g^a_{\nu\sigma}(\boldsymbol{y},\tau|\boldsymbol{x},t)$$

which satisfies the adjoint equation [7,20,24]

$$(L^a_{\mu\nu})_{y,\tau} g^a_{\nu\sigma}(\boldsymbol{y},\tau|\boldsymbol{x},t) = \delta_{\mu\sigma}\delta(\boldsymbol{x}-\boldsymbol{y})\delta(t-\tau) \tag{A1}$$

where

$$L^a_{\mu\nu} \equiv -\delta_{\nu\mu}\frac{\tilde{D}}{D\tau} - \delta_{\mu 4}\partial_\nu - \left(\frac{\widetilde{c^2}}{\gamma-1}\delta_{\nu 4} + \delta_{\nu 5}\right)\partial_\mu + K_{\nu\mu} \tag{A2}$$

and

$$\frac{\tilde{D}}{D\tau} \equiv \frac{\partial}{\partial\tau} + \tilde{v}_i(\boldsymbol{y},\tau)\frac{\partial}{\partial y_i} \tag{A3}$$

is related to the direct Green's function

$$g_{\sigma\nu}(\boldsymbol{x},t|\boldsymbol{y},\tau)$$

by the reciprocity relation

$$g^a_{\nu\sigma}(\boldsymbol{y},\tau|\boldsymbol{x},t) = g_{\sigma\nu}(\boldsymbol{x},t|\boldsymbol{y},\tau) \tag{A4}$$

Its fourth component is determined by the system [Eq. (21)] that, when written out in full, becomes [7]

$$-\frac{\tilde{D}g^a_{i4}}{D\tau} + g^a_{j4}\frac{\partial\tilde{v}_j}{\partial y_i} - \widetilde{c^2}\frac{\partial g^a_{44}}{\partial y_i} - \frac{\gamma-1}{\bar{\rho}}\frac{\partial\tilde{\theta}_{ij}}{\partial y_j}g^a_{44} - \frac{\partial g^a_{54}}{\partial y_i} = 0 \tag{A5a}$$

$$-\frac{\tilde{D}g^a_{44}}{D\tau} - \frac{\partial g^a_{i4}}{\partial y_i} + (\gamma-1)g^a_{44}\frac{\partial\tilde{v}_j}{\partial y_j} = \delta(\boldsymbol{x}-\boldsymbol{y})\delta(t-\tau) \tag{A5b}$$

$$-\frac{\tilde{D}g^a_{54}}{D\tau} - \frac{1}{\bar{\rho}}\frac{\partial\tilde{\theta}_{ij}}{\partial y_j}g^a_{i4} = 0 \tag{A5c}$$

Then, when $\boldsymbol{x},\boldsymbol{y}\to\infty$, $\widetilde{c^2}\to c_0^2 = \text{const}$ and

$$-\frac{\partial g^a_{i4}}{\partial\tau} - c_0^2\frac{\partial g^a_{44}}{\partial y_i} = 0 \tag{A6a}$$

$$-\frac{\partial g^a_{44}}{\partial\tau} - \frac{\partial g^a_{i4}}{\partial y_i} = \delta(\boldsymbol{x}-\boldsymbol{y})\delta(t-\tau) \tag{A6b}$$

$$\frac{\partial g^a_{54}}{\partial\tau} = 0 \tag{A6c}$$

which shows that g^a_{44} satisfies the inhomogeneous wave equation

$$-\frac{\partial^2 g^a_{44}}{\partial\tau^2} + c_0^2\frac{\partial^2 g^a_{44}}{\partial y_i\partial y_i} = \delta(\boldsymbol{x}-\boldsymbol{y})\frac{\partial}{\partial\tau}\delta(t-\tau) \tag{A7}$$

The relevant solution that satisfies the causality condition

$$g^a_{44}(\boldsymbol{y},\tau|\boldsymbol{x},t) = 0 \quad \text{for } t<\tau \tag{A8}$$

is

$$g^a_{44}(\boldsymbol{y},\tau|\boldsymbol{x},t) = \frac{1}{4\pi|\boldsymbol{x}-\boldsymbol{y}|c_0^2}\frac{\partial}{\partial\tau}\delta\left(\tau - t + \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_0}\right) \tag{A9}$$

So that when $x\equiv|\boldsymbol{x}|\to\infty$ with $\boldsymbol{y}$ fixed

$$g_{44}^{a} \to \frac{1}{4\pi x c_0^2} \frac{\partial}{\partial \tau} \delta\left(\tau - \frac{\boldsymbol{x} \cdot \boldsymbol{y}}{x c_0} - t + \frac{x}{c_0}\right) \tag{A10}$$

Notice that the far-field adjoint Green's function can be calculated directly by allowing $x \equiv |\boldsymbol{x}| \to \infty$ in Eq. (A7).

Appendix B: Time Average Stress

Because the subscale motion in the downstream region is likely to behave incompressibly [6], we neglect density variations in this appendix and set $\tilde{\cdot} = \hat{\cdot}$. Because the subscale stress model is specified independently of the choice of filter in most LES computations, we are at liberty to choose the implied filter more or less arbitrarily and can even choose it to be a general space–time filter [11]. We therefore define it by [17]

$$\hat{f} \equiv \bar{f} + \langle f \rangle - \overline{\langle f \rangle} = \bar{f} + \langle f'' \rangle \tag{B1}$$

where $f'' \equiv f - \bar{f}$ denotes the fluctuating component of f. Recall that

$$\sum_n a_n \langle \cdot \rangle_n$$

will be a filter when the $\langle \cdot \rangle_n$ are filters and the constants a_n sum to unity, that the filter of a filter is a filter and that the time average commutes with spatial filtering. It follows that

$$\bar{\hat{f}} = \bar{f} \tag{B2}$$

and, therefore, in view of Eqs. (9) and the following paragraph, that

$$\overline{\rho v_i' v_j'(\boldsymbol{y})} - \overline{\hat{\rho}(\widetilde{v_i v_j} - \tilde{v}_i \tilde{v}_j)(\boldsymbol{y})} = \rho(\overline{v_i' v_j'} - \overline{v_i v_j} + \overline{\hat{v}_i \hat{v}_j}) = -\rho(\overline{\hat{v}_i v_j'} + \overline{\hat{v}_i v_j'}) = -\rho\overline{[(\hat{v}_i - \bar{\hat{v}}_i)(v_j' - \bar{v}_j') + (\hat{v}_j - \bar{\hat{v}}_j)(v_i' - \bar{v}_i')]} \tag{B3}$$

which is just the sum of the covariances of $\hat{v}_i$ with v_j' and of $\hat{v}_j$ with v_i' and should therefore vanish when these quantities are uncorrelated.

Now, as explained previously, they cannot be completely uncorrelated in any actual realization of the downstream flow, because the filtered and subfilter scales will be strongly correlated near the filter cutoff. But, as argued by Kolmogorov [18], the small-scale motion should be decorrelated from the large energy-containing scales at high Reynolds numbers. We, therefore, expect that

$$\overline{\sigma_{ij}'(\boldsymbol{y})} \approx \overline{\tilde{\sigma}_{ij}(\boldsymbol{y})} \tag{B4}$$

when the cutoff for the filter $\langle \cdot \rangle$ is within the inertial subrange. The result should be exact in the present context, in which the subscale motion is determined from an actual realization of the flow, and therefore uncorrelated with $\hat{v}_i$. Equation (B4) should also hold for

$$\overline{\sigma_{4j}'(\boldsymbol{y})}$$

and

$$\overline{\tilde{\sigma}_{4j}(\boldsymbol{y})}$$

to the extent that v^2 is small compared with h.

We can therefore choose the undefined function of position in Eqs. (13) and (14) to be the common value of these two quantities and thereby obtain

$$\overline{\sigma_{\mu j}'(\boldsymbol{y})} \approx \overline{\tilde{\sigma}_{\mu j}(\boldsymbol{y})} \approx 0 \tag{B5}$$

when $\sigma_{\mu j}'$ and $\tilde{\sigma}_{\mu j}$ are defined in this fashion. This means that they are pure fluctuating quantities, which is an appropriate requirement to impose on a true sound source.

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